Collective excitations in graphene in magnetic field

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Collective excitations in graphene monolayer are studied. Equations describing collective properties of electrons in graphene are obtained. The basic ideas of the method of many-particle quantum hydrodynamics are used for the derivation. As starting point of the derivation we use the Dirac equation for massless electrons which is usually used for description of electrons in graphene [D. E. Sheehy and J. Schmalian, Phys. Rev. Lett. **99**, 226803 (2007)], where the Coulomb interaction is taken into account. We study dispersion properties of collective excitations by means derived here graphene quantum hydrodynamics equations (GQHD). We consider graphene in the external magnetic field which directed at an angle to the graphene sample. We do it in a linear approximation of the GQHD equations. We observe that the magnetic field directed perpendicular to the graphene plane had no influence on dispersion of the collective excitations. For the magnetic field directed at an angle to the graphene we obtain dependence of wave dispersion on system parameters: strength of magnetic field, wave vector, direction of wave propagation relatively to the magnetic field.

I. INTRODUCTION

Unusual properties of graphene conductivity [1], [2] have led to its widespread use in semiconductor geterostructures. Important characteristic of three dimensional (3D) and low dimensional conductors and semiconductors is a dispersion of collective excitations, particularly the plasma waves or plasmons. Knowledge of the plasmon dispersion allows us to analyze various processes in geterostructures. A model describing the collective properties of graphene is constructed in this paper. We use this model for definition of dispersion of the quantum collective excitations in graphene at the presence of the external magnetic field. The background of this research is the model presented in Ref. [3]. This model describes the dynamic of electrons in graphene and reflects essential characteristics of the conduction electrons in graphene. The basic equation describing microscopic dynamics of the conduction electrons in graphene is the many-particle equation which can be presented in the form analogous to the Schrodinger equation (or the Dirac equation) [3]

$$\imath\hbar\partial_t\psi = \hat{H}\psi, \tag{1}$$

where $\hat{H}$ is the Hamilton operator. In the absence of external fields and interaction between particles the Hamiltonian has form $\hat{H} = \sum_{i=1}^{N} v_F\sigma_i^\alpha\hat{p}_i^\alpha$, here N is the number of particles in the system, v_F is the Fermi velocity of electrons in graphene, $\hat{p}_i^\alpha$ is the momentum operator, $\hat{p}_i^\alpha = -\imath\hbar\partial_i^\alpha$, ∂_i^α is the derivative on coordinate of i-th particle, σ_i^α are the Pauli matrixes. Described model corresponds to the massless behavior of the conductivity electrons in graphene [4]. The Coulomb interaction between the electrons in graphene is considered in Ref. [3]. Equation (1) as the many-particle Schrodinger equation is not always suitable for description of the collective properties in many-particle systems. This problem connects with the fact that equation (1) determines the wave function in the 3N (2N, for two dimensional system) dimensional configuration space, whereas the collective process realize in the 3D (2D) physical space. Therefore, it is important to construct a method in the 3D (2D) physical space [5]. The method of quantum hydrodynamics (QHD) solves the problem of transition from configuration space to physical space. This method was suggested for many-particle system and has been developed for a wide class of physical systems. Method of the many-particle QHD (MPQHD) allows us derive equations for quantum observable values evolution in the 3D (2D) physical space. Equations of continuity, momentum balance, energy balance, momentum evolution (for particles with spin) and polarization (for polarized particles) are appeared from the Schrodinger equation. Most of them are analogous to classic hydrodynamic equations. In connection with described above the method is called "Quantum hydrodynamics". Method of the QHD is a modern powerful method for studying of the collective properties in systems of charged [6]- [10], and neutral [11], [13] particles (see also review [14]). Unusual properties of electrons of conductivity in graphene mapped in the equation (1) and mapped in the form of the QHD equations for graphene (GQHD), which we present below. The equations of the GQHD in this paper has the extreme difference from the QHD equations obtained previously [6] - [14]. In context of using of the Dirac equation for description of graphene electrons we note that in Ref. [15] the QHD equation were derived from the bispinor Dirac equation for massive particles. Obtained there equations were averaged on ensemble for receiving of equations for many-particle system. Comparison of the QHD equations for graphene with the usual QHD equations [10], [14] we present below during the derivation of the GQHD equations for graphene. As expected unusual properties of electrons of conductivity in graphene lead to exotic spectrum of elementary excitations. In 2D electron gas (2DEG) the basic type of the collective excitations is the Langmuir waves,

the dispersion dependence of these waves are

$$\omega^2 = \frac{2\pi e^2 n_0 k}{m}$$

where e, m are the charge and mass of electrons (e we consider as algebraic quantity, $e = -|e|$, for electrons), n_0 is the 2D concentration of electrons, k is the absolute value of the wave vector. For the graphene electrons in the absence of external fields the dispersion equation is

$$\omega = kv_F.$$

So, we did not obtain the contribution of interaction in dispersion, and, consequently, from our description the graphene Langmuir frequency (see Ref. [1] p.414) does not follow.

The recent studies of excitations in graphene being in a magnetic field have employed the picture of single electron excitation and it's transition between energy levels, such as the Landau levels, which exist for single charged particle in an external field. Consideration of microscopic picture of excitations show that some of these excitations lead to the formation of collective excitations [16]- [19].

If we want to study macroscopic collective excitations in the graphene, we do not need to consider one electron excitation. For described purpose we can use hydrodynamics or kinetics methods as it has been used for two dimensional electron gas, usual three dimensional carriers and semiconductors. These method get used to be used for getting of characteristics of the collective excitations in described objects, such as the Langmuir waves, the magneto-sound waves, the Bernstein modes, and etc. So, we can see that obtaining of the hydrodynamics description of graphene is the important task. The method of the many-particle QHD gives us possibility to derive GQHD equations, which make more rich a set of theoretical tools of graphene excitations studying.

Our paper is organized as follows. In Sec. II we present derivation of the GQHD. In Sec. III we describe the method for calculation of the dispersion of the collective excitations. In Sec. IV dispersion of collective excitations for the graphene electrons in the external magnetic field is studied. In Sec. V the brief description of obtained results is presented.

II. CONSTRUCTION OF THE MODEL

We derive basic equations by means of the MPQHD method [6], [10]. We use the many-particle spinor massless Dirac equation [3], [4]

$$\imath\hbar\partial_t\psi = \left(\sum_i\left(v_F\sigma^\alpha D_i^\alpha + e_i\varphi_{i,ext}\right) + \sum_{i,j\neq i}\frac{1}{2}e_ie_jG_{ij}\right)\psi \quad (2)$$

The following designations are used in the Hamiltonian (2): $D_i^\alpha = -\imath\hbar\partial_i^\alpha - e_iA_{i,ext}^\alpha/c$, $\varphi_{i,ext}$, $A_{i,ext}^\alpha$ - is the potentials of the external electromagnetic field, $\mathbf{E}_{i,ext} = -\nabla\varphi_{i,ext} - \partial_t\mathbf{A}_{i,ext}$ is the electric field, $\mathbf{B}_{i,ext} = curl\mathbf{A}_{i,ext}$ is the magnetic field, quantities e_i, m_i-are the charge and mass of particles, $\hbar$-is the Planck constant, and $G_{ij} = 1/r_{ij}$, - is the Green functions of the Coulomb interaction. Replacing $-\imath\hbar\partial_i^\alpha$ by D_i^α were used in Ref. [20] (see. p.127). It was made for account of external magnetic field In equation (2) the spinor wave function $\psi = \psi(R,t)$ depend on 2N coordinates $R = [\mathbf{r}_1, ..., \mathbf{r}_N]$ and time, where $\mathbf{r}_i = [x_i, y_i]$ is the 2D coordinates of each particle. Potentials $\varphi_{i,ext} = \varphi_{ext}(\mathbf{r}_i, t)$, $A_{i,ext}^\alpha = A_{ext}^\alpha(\mathbf{r}_i, t)$ also depend on 2D variables. This fact has deep consequences. Potential part of electric field connected with the scalar potential via space derivative: $\mathbf{E}_i = -\nabla_i\varphi_i$. Consequently in equation (2) there is no contribution of external electric field directed perpendicular to the graphene plane E_z (Contribution of E_z might appear via $\partial_t A_z$). Physically, there is no limitation on attendance of z projection of electric field and it's action on graphene electrons. Especially if the graphene sample is the part of the geterostructure or spin-field-effect transistor [1], [21], [22], [23], there exist the contribution of external electric field in normal direction to the graphene plane. The magnetic field vector to be

$$\mathbf{B} = curl\mathbf{A} = \mathbf{e}_x(\partial_yA_z - \partial_zA_y)$$
$$+\mathbf{e}_y(\partial_zA_x - \partial_xA_z) + \mathbf{e}_z(\partial_xA_y - \partial_yA_x),$$

two component of the vector potential of the magnetic field A_x, $A_y$are presented in Hamiltonian (2), and they does not depend on coordinate z. Therefore equation (2) contain z component of the magnetic field only. In this paper we interested in action of the external magnetic field directed at angle of graphene plane. Therefore, we generalized GQHD equations including whole vector of magnetic field $B\cdot e_z \to \mathbf{B} = [B_x, B_y, B_z]$.

We assumed that the Pauli matrices satisfy the following commutation relation:

$$[\sigma_i^\alpha, \sigma_j^\beta] = 2\imath\delta_{ij}\varepsilon^{\alpha\beta\gamma}\sigma_i^\gamma. \quad (3)$$

Graphene is the 2D structure and electrons of graphene are located in the plane. As we describe above, in 2D case the electrons has two coordinate x and y, but spin of the electrons can be directed in all direction, particularly, in direction of z axes, perpendicular to the graphene plane. This fact is accounted by formula (3). Two projection of spin operator are contained in the Hamiltonian (2), these are $\widehat{\sigma}_x$ and $\widehat{\sigma}_y$. Spin operator projection on z axis is appeared during derivation of the GQHD equations due to commutation relation (3).

Now, we present the derivation of model of the electron collective dynamics in graphene. As we mention above we describe the derivation of the MPQHD equations from equation (2). The first step is we present the definition for density of probability for the conduction electron system in the physical space.

$$n(\mathbf{r},t) = \sum_s\int dR\sum_i\delta(\mathbf{r}-\mathbf{r}_i)\psi^*(R,t)\psi(R,t) \quad (4)$$

where $dR = \prod_{p=1}^{N} d\mathbf{r}_p$.

The quantity $n(\mathbf{r},t)$ can be considered as the 2D concentration of the conductivity electrons. For studying of the time evolution of the concentration we differentiate the concentration (4) with respect to time and use equation (2). In the result, we receive an equation which has the form of the continuity equation:

$$\partial_t n(\mathbf{r},t) + \nabla \mathbf{j}(\mathbf{r},t) = 0, \tag{5}$$

where

$$\mathbf{j}(\mathbf{r},t) = \sum_s \int dR \sum_i \delta(\mathbf{r}-\mathbf{r}_i)\frac{1}{2}v_F \times$$

$$\times \left(\psi_s^*(R,t)\left(\sigma_i^\alpha \psi\right)_s (R,t) + h.c. \right) \tag{6}$$

and $\mathbf{j}(\mathbf{r},t) = v_F \mathbf{S}(\mathbf{r},t)$.

The quantity $\mathbf{S}(\mathbf{r},t)$ describes the spin density of the particle system. Consequently, we have equation

$$\partial_t n(\mathbf{r},t) + v_F \nabla \mathbf{S}(\mathbf{r},t) = 0. \tag{7}$$

Coordinate vector $\mathbf{r}$ has only two component. Consequently, equation (7) contains two component of the spin density vector $\mathbf{S}$, these are S_x and S_y. Our next step in construction of the model of collective motion is obtaining of an equation for spin evolution. For this aim we differentiate the quantity $\mathbf{S}(\mathbf{r},t)$ with respect to time and use equation (2). Because we known the third component of spin density vector we can study the evolution of all components of this vector. Therefore, we derive equation of evolution for $\mathbf{S} = [S_x, S_y, S_z]$. On this way we have equation of the spin evolution:

$$\partial_t S^\alpha(\mathbf{r},t) + v_F \partial^\alpha n(\mathbf{r},t) = -\frac{2}{\hbar}\varepsilon^{\alpha\beta\gamma} J_M^{\beta\gamma}(\mathbf{r},t) \tag{8}$$

Here new physical quantity is arisen: $J_M^{\alpha\beta}(\mathbf{r},t)$. The evident form of $J_M^{\alpha\beta}(\mathbf{r},t)$ is

$$J_M^{\alpha\beta}(\mathbf{r},t) = v_F \sum_s \int dR \sum_i \delta(\mathbf{r}-\mathbf{r}_i)\frac{1}{2}\times$$

$$\times \left(\psi_s^*(R,t)\left(\sigma_i^\alpha D_i^\beta \psi\right)_s (R,t) + h.c. \right). \tag{9}$$

This is the tensor of spin current. We study the 2D structures existing in 3D space. It leads to existence of z component of vectors for several physical quantities, as for the spin density vector. The spin current $J_M^{\alpha\beta}(\mathbf{r},t)$ is defined via two-vectors (9). These are the vector of spin σ^α and derivative vector operator D_i^β, the last one is the vector with two components. Therefore, we have the tensor $J_M^{\alpha\beta}(\mathbf{r},t)$ whose first index take three value x, y, z, but the second index take two value x, y. We have obtained two equation of the GQHD, it is equations (7) and (8). These equations significantly vary from the first two equations of the usual QHD [7]- [10]. In the usual QHD as in the classic hydrodynamics the first equation is the continuity equation where described the changing of number of particles in vicinity of the point of the physical space in consequence of the particles current. Instead of that we obtain the connection of particles number changing and the spin density (7).

We have obtained the equation (8) instead of the momentum balance equation (the Euler equation) in the usual QHD. The last one accounts the influence of particles interaction as each other and with the external fields. Equation (8) does not contain information about the interaction. Equation (8) differs from the usual equation of the spin evolution (the Bloch equation) which contains the vector product of the spin density and magnetic field whose leads to the spin evolution.

At this step we have no closed set of two equations (4) and (8). Thereto, received equations do not contain information about interaction. Thus, basic model, presented by equation (2), contains the Coulomb interaction between electrons. Therefore, we have the series of equations which is an analogous to the BBGKI series [24], [25] in the classic physical kinetics. We can obtain next equation of the series, namely equation of evolution for the tensor $J_M^{\alpha\beta}(\mathbf{r},t)$. This equation to be

$$\partial_t J_M^{\alpha\beta}(\mathbf{r},t) + v_F \partial^\alpha J^\beta(\mathbf{r},t)$$

$$-\hbar v_F^2 \varepsilon^{\alpha\gamma\delta}\partial^\beta\partial^\gamma S^\delta(\mathbf{r},t) = -\frac{e}{c}v_F^2\varepsilon^{\alpha\beta\gamma}n(\mathbf{r},t)B^\gamma(\mathbf{r},t)$$

$$+\frac{2v_F^2}{\hbar}\varepsilon^{\alpha\mu\nu}\Pi^{\nu\mu\beta}(\mathbf{r},t) + ev_F S^\alpha(\mathbf{r},t)E^\beta(\mathbf{r},t)$$

$$- e^2 v_F S^\alpha(\mathbf{r},t)\partial^\beta \int d\mathbf{r}' G(\mathbf{r},\mathbf{r}')n(\mathbf{r}',t). \tag{10}$$

Equation (10) contains interaction and two new quantities, these are $J^\alpha(\mathbf{r},t)$ and $\Pi^{\alpha\beta\gamma}$. They are

$$J^\alpha(\mathbf{r},t) = v_F \sum_s \int dR \sum_i \delta(\mathbf{r}-\mathbf{r}_i)\frac{1}{2}\times$$

$$\times \left(\psi_s^*(R,t)\left(D_i^\alpha \psi\right)_s (R,t) + h.c. \right) \tag{11}$$

and

$$\Pi^{\alpha\beta\gamma}(\mathbf{r},t) = \sum_s \int dR \sum_i \delta(\mathbf{r}-\mathbf{r}_i)\frac{1}{2}\times$$

$$\times \left(\psi_s^*(R,t)\left(\sigma_i^\alpha D_i^\beta D_i^\gamma \psi\right)_s (R,t) + h.c. \right). \tag{12}$$

Equation (10) is the first equation in system of the GQHD equations which contain the magnetic field. In equation (10) we have made generalization and consider all

three component of the magnetic field instead of B_z which presented in the Hamiltonian (2).

The second rank tensor $\varepsilon^{\alpha\beta\gamma}B^{\gamma}_{ext}$ contains strength of the external magnetic field and gives contribution to the evolution of spin current. B_x and B_y do not include in equation (2), but they can make influence on particles dynamic.

Quantity $\mathbf{J}(\mathbf{r},t)$ is an analog of the current of probability or the momentum density in the usual QHD [6], [14], [10]. This is very important physical quantity and we propose that our model must be complete by one more equation - the momentum balance equation. For derivation of the momentum balance equation, i.e. equation of evolution of $\mathbf{J}(\mathbf{r},t)$, we apply the same method as used above. We differentiate $\mathbf{J}(\mathbf{r},t)$ with respect to time and use the equation (2). In the result we have

$$\partial_t J^{\alpha}(\mathbf{r},t) + v_F \partial^{\beta} J^{\beta\alpha}_{M} = \frac{e v_F^2}{c}\varepsilon^{\alpha\beta\gamma}S^{\beta}B^{\gamma}$$

$$+ e v_F n E^{\alpha} - e^2 v_F n \partial^{\alpha}\int d\mathbf{r}' G(\mathbf{r},\mathbf{r}')n(\mathbf{r}',t). \qquad (13)$$

Equation (13) is very similar to the Euler equation in the usual QHD. But instead of the Lorentz force we have vector product of the spin density and the magnetic field.

Equation (13), just as equation (10), contains interaction between particles. In fact equation (13) is an analog of the Euler equation in the usual QHD [6], [14], [10]. Obtained set of equations (7), (8), (10) and (13) is not close because the set of equations contains quantity $\Pi^{\alpha\beta\gamma}(\mathbf{r},t)$. We can obtain equation for $\Pi^{\alpha\beta\gamma}(\mathbf{r},t)$, but in this way we will obtain new physical quantities. We need to make a closed set of the equations (7), (8), (10) and (13), expressing $\Pi^{\alpha\beta\gamma}(\mathbf{r},t)$ via physical quantities included in these equations. For closing the set of equations (7), (8), (10) and (13) we use method of approximate calculations of the hydrodynamic variables developed in [6], [11]. Following the paper [6], [11] we obtain

$$\Pi^{\alpha\beta\gamma}(\mathbf{r},t) = S^{\alpha}(\mathbf{r},t)\frac{J^{\beta}(\mathbf{r},t)J^{\gamma}(\mathbf{r},t)}{n^2(\mathbf{r},t)}$$

$$+ \varrho^{\alpha\beta\gamma}(\mathbf{r},t) - \hbar^2 S^{\alpha}(\mathbf{r},t)\frac{\partial^{\beta}\partial^{\gamma}\sqrt{n(\mathbf{r},t)}}{\sqrt{n(\mathbf{r},t)}}. \qquad (14)$$

where $\varrho^{\alpha\beta\gamma}(\mathbf{r},t)$ describes the contribution of the thermal motion, so, it is an analog of the tensor of kinetic pressure in the usual hydrodynamics. In this paper we do not account the thermal motion. Consequently we suggest that $\varrho^{\alpha\beta\gamma}(\mathbf{r},t)$ is equal to zero. The last term in formula (14) is an analog of the quantum Bohm potential (see for example [10]).

Now, we have a closed system of the equations (7), (8), (10), (13) and (14). Using the GQHD equations (7), (8), (10), (13) and (14) we can study the collective excitations and they properties, i.e. dispersion dependence and increments of instabilities.

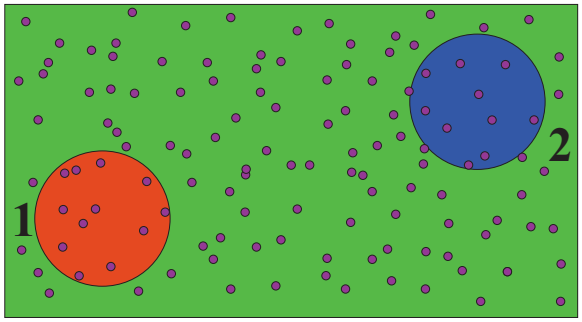

FIG. 1. The figure presents the picture of the self-consistent interaction in the system of charged particles.

Developing method allows us to derive the energy balance equation which give us possibility to study the influence of the temperature on the graphene dynamics.

Self-consistent field approximation: discussion

In last terms of equations (10) and (13) we have used the self-consistent field approximation. Here we present general expressions for two-particle functions appeared in equations (10) and (13), and also describe the meaning of the self-consistent approximation. This approximation was suggested by A. A. Vlasov in 1938 [26] for many-particle system of charged particles.

The Fig. 1 presents the picture of the self-consistent interaction between charges particles. Total charge of the region 2 interacts with the total charge of the region 1. Changing the extreme point of a radius vector (or shifting the region 2) we can scan whole space. In this way we obtain action of the external charges on the region 1. Changing position of the region 1 and repeating described operation we obtain action surrounding charges on each region of space. This is a picture of the self-consistent interaction in fixed moment of time and this picture governs an evolution of particles in system. This picture of interaction is typical for the classic physics, where we need to obtain smooth functions describing the collective motion. For that is necessary to average at physically infinitesimal volume (sketched circle). In the quantum mechanics, the concentration, the spin density, the current density, etc, are defined via wave function and we can consider described picture on interaction of separate particle instead of the space regions.

In general case the last terms in the equations (10) and (13) contain the two-particle function which in the self-consistent field approximation expressed via one particle functions and in this approximation we obtain a closed set of equations. In the equations (10) and (13) the following two-particle functions were appeared. Two-particle concentration

$$n_2(\mathbf{r},\mathbf{r}',t) = \sum_s \int dR\times$$

$$\times \sum_{i,j\neq i} \delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j)\psi_s^+(R,t)\psi_s(R,t) \qquad (15)$$

is the average of the product of the concentration operators $\sum_{i=1}^{N}\delta(\mathbf{r}-\mathbf{r}_i)$, where N is the number of particles in the system. The quantity

$$j_2^\alpha(\mathbf{r},\mathbf{r}',t) = \sum_s \int dR$$

$$\times \sum_{i,j\neq i} \delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{r}'-\mathbf{r}_j)v_F\psi_s^+(R,t)\left(\sigma_i^\alpha\psi\right)_s(R,t) \qquad (16)$$

is the two-particle function of the concentration and the spin density $\sum_{i=1}^{N}\sigma_i^\alpha\delta(\mathbf{r}-\mathbf{r}_i)$. Technically, the self-consistent field approximation corresponds to the factoring of the two-particle functions in product of one-particle ones:

$$n_2(\mathbf{r},\mathbf{r}',t) \to n(\mathbf{r},t)n(\mathbf{r}',t) \qquad (17)$$

and

$$j_2^\alpha(\mathbf{r},\mathbf{r}',t) \to v_F S^\alpha(\mathbf{r},t)n(\mathbf{r}',t). \qquad (18)$$

It was used at derivation of the equations (10) and (13). This approximation is suitable at consideration of the long-range interaction, particularly for the Coulomb interaction.

If we interest in more detail description of interaction we need to consider many-particle correlations. For example, for the two-particle concentration the correlation to be

$$g(\mathbf{r},\mathbf{r}',t) = n_2(\mathbf{r},\mathbf{r}',t) - n(\mathbf{r},t)n(\mathbf{r}',t),$$

where $g(\mathbf{r},\mathbf{r}',t)$ includes a quantum correlation caused by exchange interaction. A method of correlation calculation was developed in Ref.s [6], [11] and [27]. This method also can be used for the graphene description, and for it's further and more detailed studying.

III. METHOD OF CALCULATION OF WAVE DISPERSION

We consider the small perturbation of the equilibrium state like

$$n = n_0 + \delta n,\ \ S^\alpha = S_0^\alpha + \delta S^\alpha,\ \ \mathbf{S}_0 \parallel \mathbf{B}_0$$

$$J^\alpha = 0 + \delta J^\alpha, J_M^{\alpha\beta} = 0 + \delta J_M^{\alpha\beta}, \qquad (19)$$

where $\mathbf{B}_0$ is the external magnetic field, $\mathbf{B}_0 = [B_{0x}, 0, B_{0z}]$.

Substituting these relations into system of the GQHD equations (7), (8), (10), (13) and (14) *and* neglecting by nonlinear terms, we obtain a system of linear equations with constant coefficients.

Passing to the following representation for small perturbations δf

$$\delta f = f(\omega,\mathbf{k})exp(-\imath\omega t + \imath\mathbf{kr})$$

yields the homogeneous system of algebraic equations

$$-\imath\omega\delta n + \imath v_F\mathbf{k}\delta\mathbf{S} = 0, \qquad (20)$$

$$-\imath\omega\delta S^\alpha + \imath v_F k^\alpha\delta n = -\frac{2}{\hbar}\varepsilon^{\alpha\beta\gamma}\delta J_M^{\beta\gamma}, \qquad (21)$$

$$-\imath\omega\delta J^\alpha + \imath v_F k^\beta\delta J^{\beta\alpha} = \frac{ev_F^2}{c}\varepsilon^{\alpha\beta\gamma}B_0^\gamma\delta S^\beta + \imath k^\alpha e^2 v_F n_0(2\pi/k)\delta n \qquad (22)$$

and

$$-\imath\omega\delta J_M^{\alpha\beta} + \imath v_F k^\alpha\delta J^\beta + \hbar v_F^2 k^\beta k^\gamma\varepsilon^{\alpha\gamma\delta}\delta S^\delta = -\frac{e}{c}v_F^2\varepsilon^{\alpha\beta\gamma}B_0^\gamma\delta n + v_F^2\hbar\varepsilon^{\alpha\mu\nu}\frac{1}{n_0}S_0^\nu k^\mu k^\beta\delta n + \imath e^2 v_F S_0^\alpha k^\beta(2\pi/k)\delta n. \qquad (23)$$

We assume that the spin density magnitude has a nonzero value. Expressing all quantities entering in the set of equations in terms of the spin density, we get the equation

$$\Lambda^{\alpha\beta}(\omega,\mathbf{k})\cdot S^\beta(\omega,\mathbf{k}) = 0,$$

where

$$\Lambda^{\alpha\beta}(\omega,\mathbf{k}) = \omega^2\delta^{\alpha\beta} - v_F^2 k^\alpha k^\beta + 2\frac{e}{c}\frac{1}{\hbar\omega}\varepsilon^{\alpha\gamma\mu}\varepsilon^{\gamma\delta\nu}B_0^\nu k^\beta k^\delta k^\mu\frac{v_F^5}{\omega^2 - v_F^2k^2}$$

$$-2\frac{e}{\hbar c}\varepsilon^{\alpha\gamma\mu}\varepsilon^{\beta\gamma\nu}B_0^\nu k^\mu\frac{\omega v_F^3}{\omega^2 - v_F^2k^2} + 2\varepsilon^{\alpha\gamma\mu}\varepsilon^{\beta\delta\mu}v_F^2k^\gamma k^\delta$$

$$+2\frac{e}{c}\varepsilon^{\alpha\gamma\mu}\varepsilon^{\gamma\mu\nu}B_0^\nu\frac{1}{\hbar\omega}v_F^3k^\beta + 2\varepsilon^{\alpha\gamma\delta}\varepsilon^{\delta\mu\nu}v_F^3k^\beta k^\gamma k^\mu S_0^\nu\frac{1}{n_0\omega}$$

$$+ 4\pi\imath e^2\varepsilon^{\alpha\gamma\delta}k^\beta k^\gamma S_0^\delta\frac{1}{\hbar\omega k}. \qquad (24)$$

Contribution of the Coulomb interaction is presented by the last term in the dispersion matrix $\Lambda^{\alpha\beta}$.

In the absence of external magnetic field the dispersion matrix $\Lambda^{\alpha\beta}$ can be presented as

$$\Lambda^{\alpha\beta}(\omega,\mathbf{k}) = \omega^2\delta^{\alpha\beta} - v_F^2k^\alpha k^\beta + 2\varepsilon^{\alpha\gamma\mu}\varepsilon^{\beta\delta\mu}v_F^2k^\gamma k^\delta.$$

Dispersion equation to be

$$det\widehat{\Lambda}(\omega,\mathbf{k}) = 0. \qquad (25)$$

This equation can be represent in the form

$$(\Lambda_{xx}\Lambda_{yy} - \Lambda_{xy}\Lambda_{yx})\Lambda_{zz} = 0, \qquad (26)$$

and splits into two

$$\Lambda_{xx}\Lambda_{yy} - \Lambda_{xy}\Lambda_{yx} = 0, \qquad (27)$$

and

$$\Lambda_{zz} = 0. \qquad (28)$$

IV. GRAPHENE IN THE MAGNETIC FIELD

In this section we consider dispersion properties of waves in graphene placed in the external magnetic field which parallel or perpendicular to the XY plane, we suppose that graphene is located in the XY plane.

We begin this chapter with the consideration of equation

$$\Lambda_{xx}\Lambda_{yy} - \Lambda_{xy}\Lambda_{yx} = 0. \tag{29}$$

For the magnetic field perpendicular to the graphene plane $\mathbf{B}_0 = B_z\mathbf{e}_z$ from (29) we have

$$\omega = v_F k. \tag{30}$$

It is the same result as in the absence of the external field.

Suggesting what $S_{0x} \sim B_{0x}$ we can admit that equation (29) contains terms proportional to B_{0x} and B_{0x}^2. Considering the limit of approximately small in-plane magnetic field B_{0x} we can neglect by the terms proportional to the B_{0x}^2. In the result we find an equation which is the equation of eight degree of ω. However, at the considering values of the parameters of the system we get that this equation has two solutions only. These solutions reveal strong nonmonotone dependence of the collective excitation frequency ω on the angle between direction of wave propagation and the direction of the in-plane magnetic field (2).

From equation (29) we find that in described approximation ξ depends on three dimensionless parameters

$$\kappa = \frac{eB_{0x}}{c\hbar k^2} < 0,$$

$$\mu = \frac{e^2 S_{0z}}{\hbar v_F k^2},$$

$$\chi = S_{0x}/n_0,$$

and angle of wave propagation θ. The angle θ is defined as $\cos\theta = k_x/k$ and $\sin\theta = k_y/k$.

Let's consider equation $\Lambda_{zz} = 0$. Dispersion equation has following explicit form

$$\omega^2 + 2v_F^2 k^2 - 2\frac{e}{\hbar c}\frac{\omega v_F^3 \mathbf{kB}_0}{\omega^2 - v_F^2 k^2} = 0 \tag{31}$$

or in dimensionless form to be

$$\xi^2 + 2 - \frac{2\alpha\xi}{\xi^2 - 1} = 0 \tag{32}$$

where

$$\xi = \frac{\omega}{kv_F} \tag{33}$$

and

$$\alpha = \frac{e}{\hbar c}\frac{\mathbf{kB}}{k^3} = -\frac{|e|}{\hbar c}\frac{B_0}{k^2}\cos\theta. \tag{34}$$

ξ describe the frequency of the wave divided by kv_F, α present the contribution of the external magnetic field

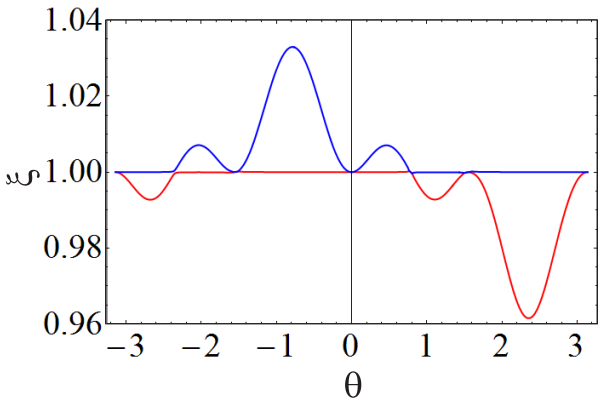

FIG. 2. The figure presents the dependence of the reduced frequency ξ (33), appearing from equation (29) in presence of the in-plane magnetic field B_{0x}, on the angle θ, which is the angle between axes Ox and direction of the wave propagation. Dimensionless parameters of the system are choose to be equal to the following quantities $\kappa = -10^{-6}$, $\chi = 0.1$, and $\mu = 10^{-3}$. Blue curve (upper) lying at $\xi \geq 1$ corresponds to one of solutions of equation (29) and red curve (lowest) lying at $\xi \leq 1$ described the second solution.

which parallel to the graphene plane and θ is the angle between the magnetic field and the direction of the excitation propagation.

In the absence of the external field or if the magnetic field directed perpendicular to the graphene plane from (32) we have

$$\xi^2 + 2 = 0,$$

thus the equation $\Lambda_{zz} = 0$ has no wave solution.

Here we consider equation (32) in the case when the magnetic field parallel to the graphene plane, then the equation (32) we represent in the form

$$\xi^4 + \xi^2 - 2\alpha\xi - 2 = 0. \tag{35}$$

The dependence of ξ on α is presented Fig. 3. This solution shows no instabilities. We obtain this solution from the equation

$$\Lambda_{zx}S_x + \Lambda_{zy}S_y + \Lambda_{zz}S_z = 0, \tag{36}$$

since $\Lambda_{zz} = 0$ and $S_z \neq 0$ we can note that obtained wave solution contains wave of spin, where amplitude of spin density directed perpendicular to the graphene plane and, thus, to the direction of the wave propagation. We need to pay attention to the fact that the Coulomb interaction contained in the basic equations and the dispersion matrix does not influence on found dispersion dependence.

V. CONCLUSION

We investigated the influence of the external static uniform magnetic fields on the dispersion properties of linear waves of the graphene electrons. We supposed that

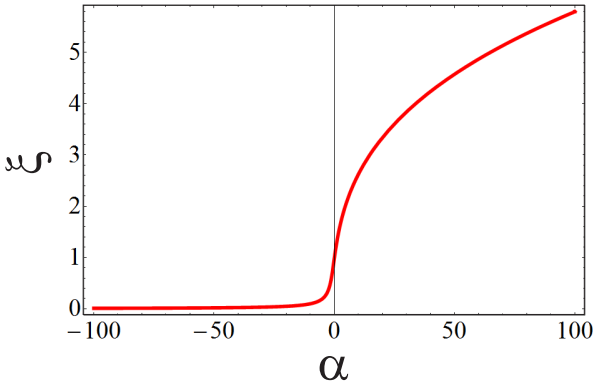

FIG. 3. The figure presents the dependence of the reduced frequency ξ (33) on parameter α (34).

the external field directed at an angle to the plane where graphene is located. We paid attention to the particular cases when the angle between the magnetic field direction and the graphene plane equal to 0 or $\pi/2$. In the absence of external fields the dispersion dependence of the collective excitation has form $\omega = v_F k$ (see formula (30)). If the magnetic field perpendicular to the graphene plane there is no changes in the dispersion in compare with the case of graphene in the absence of the external field. We obtained dispersion dependence for the case magnetic field parallel to the graphene plane and studied dependence of the frequency of the collective excitations on strength of the external magnetic field and angle between the magnetic field and the direction of excitation propagation. We also showed that in the presence of an in-plane magnetic field B_{0x}, along with B_{0z}, solution $\omega = v_F k$ splits on two.

For studying of described problem we derived system of the QHD equations for electrons in graphene. For this derivation we used the method of the MPQHD. Obtained GQHD equations consist of four equations: equations of concentration evolution, spin evolution, current evolution and spin current evolution. In this equation we made generalization and included in the equations the contribution of B_x, B_y along with the B_z.

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[1] S. Das Sarma, Shaffique Adam, E. H. Hwang, Enrico Rossi, Rev. Mod. Phys. **83**, 407 (2011).
[2] N. M. R. Peres, Rev. Mod. Phys. **82**, 2673 (2010).
[3] D. E. Sheehy and J. Schmalian, Phys. Rev. Lett. **99**, 226803 (2007).
[4] K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, A. A. Firsov, Nature **438**, 197 (2005).
[5] S. Goldstein, Physics Today. **51**, N. 3, 42 (1998); S. Goldstein, Physics Today. **51**, N. 4, 38 (1998).
[6] L. S. Kuz'menkov and S. G. Maksimov, Theoretical and Mathematical Physics **118**, 227 (1999).
[7] M. Marklund and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007).
[8] G. Brodin, M. Marclund, New J. Phys. **9**, 277 (2007).
[9] P. A. Andreev, L. S. Kuzmenkov, arXiv:1109.3335.
[10] P. A. Andreev, L. S. Kuz'menkov, M. I. Trukhanova, Phys. Rev. B **84**, 245401 (2008).
[11] P. A. Andreev, L. S. Kuz'menkov, Phys. Rev. A **78**, 053624 (2008).
[12] P. A. Andreev, arXiv:1109.0896.
[13] P. A. Andreev, L. S. Kuzmenkov, arXiv:1106.0822.
[14] P. K. Shukla, B. Eliasson, Rev. Mod. Phys. **83**, 885 (2011).
[15] F. A. Asenjo, V. Munoz, J. A. Valdivia, and S. M. Mahajan, Phys. Plasmas **18**, 012107 (2011).
[16] R. Roldan, J.-N. Fuchs, and M. O. Goerbig, Phys. Rev. B **80**, 085408 (2009).
[17] R. Roldan, J.-N. Fuchs, and M. O. Goerbig, Phys. Rev. B **82**, 205418 (2010).
[18] R. Roldan, M. O. Goerbig, and J.-N. Fuchs, Phys. Rev. B **83**, 205406 (2011).
[19] M. O. Goerbig, Rev. Mod. Phys. **83**, 1193 (2011).
[20] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov and A. K. Geim, Rev. Mod. Phys. **81**, 109 (2009).
[21] L. Britnell, R. V. Gorbachev, R. Jalil, B. D. Belle, F. Schedin, M. I. Katsnelson, L. Eaves, S. V. Morozov, N. M. R. Peres, J. Leist, A. K. Geim, K. S. Novoselov, L. A. Ponomarenko, Field-effect tunneling transistor based on vertical graphene heterostructures, arXiv:1112.4999
[22] J. C. Martinez-Orozco, I. Rodriguez-Vargas, C. A. Duque, M. E. Mora-Ramos, L. M. Gaggero-Sager, Phys. Status Solidi B. **246**, 581 (2009); K. Nakagawa, A. A. van Gorkum, Y. Shiraki, Appl. Phys. Lett. **54**, 1869 (1989).
[23] I. Zutic, J. Fabian, S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
[24] I. A. Akhiezer, *Plasma electrodynamics*, (Pergamon Press, 1975).
[25] L. P. Pitaevskii, E.M. Lifshitz "Physical Kinetics" (kinetic theory), Vol. 10 of Course of Theoretical Physics (Pergamon, London, 1981).
[26] A. A. Vlasov, J. Exp. Theor. Phys. **8**, 291 (1938); A. A. Vlasov, Sov. Phys. Usp. **10**, 721 (1968).
[27] L. S. Kuz'menkov, S. G. Maksimov, and V. V. Fedoseev, Theoretical and Mathematical Physics **126**, 212 (2001).